Aspects of phase problems for multidimensional and undersampled data

Rick Millane
rick@moa.foodsci.purdue.edu

Whistler Center for Carbohydrate Research and
Computational Science and Engineering Program
Purdue University
West Lafayette, IN 47907-1160

Diffracted fields from "isolated" objects exist everywhere. In phase retrieval problems for such objects the Fourier amplitude can then be measured effectively continuously in Fourier space. The phase retrieval problem is then well-determined for two-dimensional problems (but is under-determined for one-dimensional problems). As might be expected, for problems in more than two dimensions, such phase retrieval problems are over-determined.

If the diffracting object contains periodicities, the diffracted field may vanish except on particular manifolds in Fourier space. A number of practical three-dimensional imaging problems of this kind occur. The classical example is crystallography, where the diffracted amplitude exists only on the "reciprocal lattice," a point set whose average spacing is larger than that required for unique image reconstruction from the amplitude. Other cases occur however (e.g. if the diffracting object is a planar crystalline array or a fiber) in which the Fourier amplitude can be measured on a denser manifold in Fourier space. For these cases there is a trade-off between the effects of the higher dimensionality of the problem and the degree of undersapmling in Fourier space. Some of the resulting interesting uniqueness properties and reconstruction problems will be discussed.